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Average temperature of the rotor of high-speed rotating heat exchanger during the period of thermal start-up

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Abstract

The paper presents results of the calculations of the average temperature variations of model rotor of a high-speed rotating heat exchanger in the course of thermal start up. A model rotor adopted for the calculations has a porous structure formed by radially oriented ducts of a constant circular cross-section. The scope of this paper covers numerical calculations of temperature distribution in the material of the rotor versus its rotational speed, temperature difference of the gases at the inlet, and types of the rotor material. With the use of these results, graphs are elaborated representing the dependence of the rotor's dimensionless temperature on the Fourier number, with the Biot number used as a parameter.

1. Introduction

Throughout the recent period, at the turn of the 20th century, emphasis has been put on various forms of saving of all kinds of energy and on environmental protection, and consequently, the issues related to accumulation, transformation and transportation of heat energy have gained special importance. Therefore, construction of new devices allowing for a more effective use of energy constitutes a very up-to-date problem. The present paper deals with such a device—a high-speed rotating sucking and forcing heat exchanger. Based on the traditional classification of heat exchangers, i.e. devices in which a process of heat exchange between two or more media is effected, a high-speed rotating heat exchanger may be included in the group of regenerators characterised by a transversal direction movement of the filling, and more precisely—in the group of rotational regenerators, in which the filling has a form of a porous disk or a drum. The literature concerning devices of such a type is relatively extensive. For example, one should mention paper [1], where a theory of a low-speed regenerator is described, and in particular an analytic method of calculation of stationary temperature field is presented, taking into account the heat conduction in the space with the filling treated as a material characterised by certain equivalent heat conductivity. Heat transfer in a rotational regenerator is also considered in [2] where, by means of the finite difference method, examples of temperature distribution patterns have been obtained for granular ceramic filling. Other examples are provided in papers [3] and [4], where analytical expressions have been obtained for temperature distribution patterns in the gases and fillings in a low-speed regenerator, as well as experimental verification of the mathematical model

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Nomenclature

a	thermal diffusivity, m^2/s
A	surface, m^2
Bi	$=\alpha L/\lambda_r$, Biot number
d, D	diameter, m
Fo	$=at^2/L$, Fourier number
L	characteristic dimension, m
n	rotational speed, rpm
Nu	$=\alpha d/\lambda_g$, Nusselt number
p	porosity
Re	$=wd/v$, Reynolds number
Ro	$=\omega d^2/w$, Rossby number
t	time, s
T	temperature, K, °C
V	volume, m^3
w	mean fluid velocity in the duct of the rotor, m/s
x, y, z	space coordinates

Greek symbols

α	heat transfer coefficient, W/m^2 K
λ	thermal conductivity, W/m K
v	kinematic viscosity, m^2/s
ω	angular velocity, s^{-1}
Θ	dimensionless temperature

Subscripts

H, C	hot and cold respectively
g	for gas
r	for rotor material
0	initial
1	at inlet

described has been performed. Other examples are available in paper [5] presenting a theory of modelling of a rotational mass and a heat exchanger by means of the shock wave method and the method of characteristics based on an analysis of the operation of an ideal humidifier, and in paper [6], where results of experiments on local heat transfer along the ducts constituting the porous structure of the moving filling have been presented.

All the papers mentioned above deal with low-speed regenerators and/or heat and mass exchangers, i.e. devices in which rotational velocities of the moving filling (rotor) are of the order from several to about 20 rotations per minute.

Because of its specific construction, high rotational speed of the filling (of the order of several thousand rotations per minute) and its small dimensions, the device considered in the present paper may not be qualified as a member of the family of typical rotational regenerators. A high-speed rotational heat exchanger is a combination of two flow devices—a fan and a heat exchanger. Its fundamental part consist of a cylindrical porous rotor driven from the outside, inside which there is a fixed diaphragm separating a cold gas stream from a hot one. The rotor revolving with a high speed causes suction of the hot and cold gas from the opposite sides of the exchanger, then heating and cooling of the cold and hot gas, respectively, and at last it forces these gas streams through symmetrical outlet channels of the device housing.

In the region of hot gas cooling, thermal power is being conducted to the inside of the rotor material and accumulated there, while this phenomenon is of a transient character. In the region of heating, respectively, thermal power is being collected from the rotor material.

A schematic diagram including basic construction details of such a device has been presented in [7].

As for the detailed description of a high-speed rotating sucking and forcing heat exchanger, it has been presented in papers [8,9], in which results of theoretical and experimental examinations of the device prototype are demonstrated. Among others, fan characteristics for various porous rotor structures, intensity of the forced convection heat exchange, and thermal properties of the rotor are presented there. Theoretical analyses presented in [8,9] were based on the assumption of a steady state of the operation conditions of an exchanger.

Then, another paper related also to a high-speed heat exchanger [10], presents an analysis of the operation of such a device working in a serial connection of several high-speed regenerators.

The problem of the transient exchange processes occurring in the course of the exchanger operation has been taken into account in papers [11,12], which deal with the calculations of thermal power for a high-speed rotating heat exchanger and the calculations of temperature distribution patterns of the gases flowing through a regenerator's rotor.

2. Aim and scope of this paper

The aim of this paper is to determine average temperature variations of a model rotor of a high-speed rotating heat exchanger in the course of thermal start-up. The period of thermal start-up is usually defined as the time interval measured from the moment of reaching the intended rotational speed by the rotor (a mechanical start-up) and "switching-in" of the cold and hot gas streams into the system, to the moment when the rotor reaches a quasi-stationary value of temperature, at which oscillations occur. The temperature oscillations at quasi-stationary operation are related to the cyclic transition of the rotor through the cooling and heating zones.

The scope of this paper covers numerical calculations of temperature distribution in the material of the rotor versus its rotational speed, temperature difference of the gases at the inlet, and types of the rotor material. The following numerical values have been applied in the paper:

- rotor speeds equal to 1000, 2000, 3000 and 3500 rpm,
- temperature difference of the gases at the inlet equal to 50, 100, 200, 300, 400, and 500 K, at the assumed constant temperature of the cold gas at the inlet T_{CI} = 25 °C,
- materials of the rotor: duralumin, copper, polyurethane, and porcelain.

A model rotor adopted for the calculations (Fig. 1) had a porous structure formed by radially oriented ducts of a constant circular cross-section. The diameter of the ducts equalled 1.5 mm, while external and internal diameters of the cylindrical rotor were 106 and 76 mm, respectively. The dimensions of the model rotor have been imposed by the size of the actual rotor used in the device prototype [8]. In the prototype, other porous rotor materials were applied too [8,9]. However, the adoption of the "uniform duct" structure in the present paper was dictated by the simplicity of the geometry of such a system, which was desirable in numerical modelling.

3. Mathematical formulation of the problem

The transient character of the heat transfer processes occurring in a regenerator's rotor is of a cyclical type and results from a change of the angle between the duct (pore) axis and the plane of the fixed diaphragm separating the hot gas from the cold one. The processes of heat transfer occurs at an assumed constant rotational velocity of the rotor, constant mass flow streams, and constant gas temperatures at the inlet.

Due to the simple geometrical structure of the rotor, a single element of the rotor material, adjacent to a single duct and denoted by S in Fig. 1, has been considered. For this element, according to the adopted system of coordinates, one may formulate a differential equation representing transient heat conduction in the rotor (carcass) material. Taking into account the simplified assumptions after-mentioned, 2D equation has been considered

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \qquad (1)$$

3.1. Assumptions made to simplify the model

In the mathematical model of heat transport in the rotor material, a two-dimensional dependence of the temperature field has been assumed, while conduction

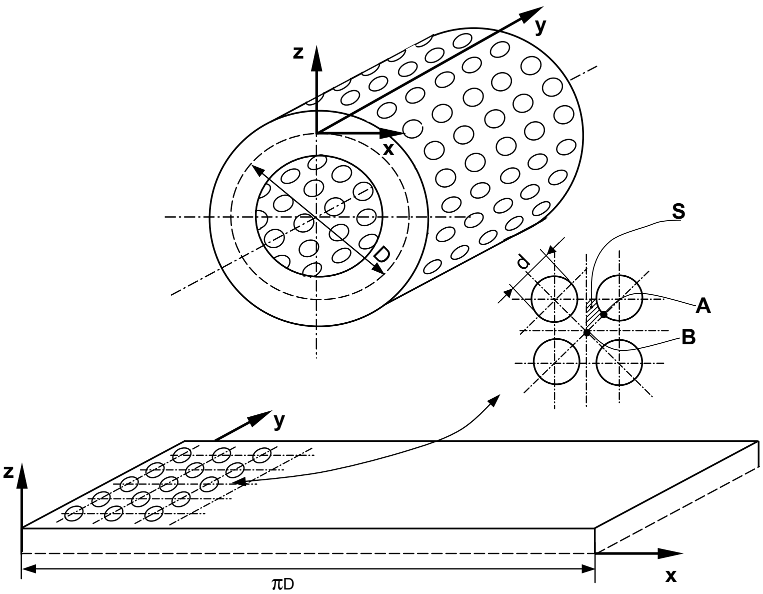


Fig. 1. Development of the rotor cylindrical surface on its mean diameter and porous structure of the rotor.

along z-axis (along the duct) has been neglected. The differential equation of transient heat conduction in the rotor material has been formulated in area S, the dimensions of which corresponded to the average rotor diameter (Fig. 1). The assumption made to simplify the model was based on the results of previous numerical calculations concerning the exchanger's thermal power, described in [11,13]. By means of segmentation along the duct (cf. Fig. 6 in [11]) and solving the two-dimensional heat conduction equation at the inlet of each segment, taking into account gas temperature variation along a single duct, variability of the material temperature vs. z-axis was taken into account implicitly. The ducts' internal surface temperature distribution pattern obtained in this way demonstrates that the maximum temperature gradients in the material along the duct occur up to about 1/3 of the duct length, while the maximum decrease (increase) does not exceed 1 K in the case of the maximum gas temperature difference at the inlet.

It must be mentioned that accounting for the gas temperature variation along the duct was possible due to the calculations of the gas temperature distribution patterns along the duct based on the energy balance equations for the gas in particular segments [11,13].

Another simplification in the course of determination of the average rotor material (carcass) temperature in the period of thermal start-up was an assumption related to the method of averaging of this temperature. It was assumed, then, that the temperature at point B in Fig. 1, i.e. in the middle of the length of the duct, is equal to the average temperature in the whole carcass at a given time.

3.2. The calculation method

The two-dimensional differential equation of transient heat conduction in the rotor's material (1) was solved by means of the finite difference method. The space domain subject to digitisation was the area S mentioned above. On the other hand, the digitised time interval was the period of the presence of a single duct in the cooling and heating zones, successively, which resulted from the rotational speed of the rotor. The Eq. (1) was replaced by a system of difference equations:

$$\left.\begin{aligned} \left(\frac{\partial T}{\partial t}\right)_{i,j,k+1} &\approx \frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t} \\ \left(\frac{\partial^2 T}{\partial x^2}\right)_{i,j,k+1} &\approx \frac{T_{i+1,j,k+1}-2T_{i,j,k+1}+T_{i-1,j,k+1}}{(\Delta x)^2} \\ \left(\frac{\partial^2 T}{\partial y^2}\right)_{i,j,k+1} &\approx \frac{T_{i,j+1,k+1}-2T_{i,j,k+1}+T_{i,j-1,k+1}}{(\Delta y)^2} \end{aligned}\right\}, \qquad (2)$$

where i, j—number of space node, k—number of time step.

The numerical method applied was conformant with the fundamental requirements concerning numerical methods used for solving problems of physics. The difference approximation consistence was observed, i.e. the condition was fulfilled that the difference operator representing a given discrete problem must be consistent with the differential operator describing the continuous problem related to the discrete one.

An additional advantage of the method applied consists in the simplicity of the physical interpretation, especially the consideration of the boundary problem, where the finite difference method was reduced to the elementary balance method.

The replacement of the time derivative by a reverse difference quotient provided for the convergence and the stability of the created set of equations with the arbitrary selection of steps Δx and Δy in space and digitisation interval Δt in the domain of time.

The applied numerical method constituted a part of a more comprehensive self-made computational code previously used also for the calculation of thermal power of an exchanger [11,12] and its temperature effectiveness [14].

3.3. Boundary condition

One of the main conditions necessary for uniqueness of the differential equation representing the transient heat conduction in a solid body is the boundary condition representing the heat transfer conditions at the body surface. In the case considered here, the surface of the body consists of the internal surface (boundary) of a duct, through which heat is being exchanged from the gas flowing through to the rotor (carcass) material by means of convection.

In the calculation method applied, the finite difference method at the duct boundary was reduced to the elementary balance method. On the basis of this method, balance equations in the boundary nodes were created of the following general form:

$$q_a = q_\alpha + q_\lambda, \qquad (3)$$

where q_a—accumulated thermal power (temperature increase in an element); q_α—thermal power intercepted from the fluid by means of convection (boundary condition of the third type); $q_\lambda = q_{\lambda 1} + q_{\lambda 2} + q_{\lambda 3}$—conducted thermal power (according to Fig. 2—the energy balance of the element with dimensions $\frac{\Delta x}{2} \times \Delta y \times 1$ in time interval Δt).

Adequate equations have a form (according to Fig. 2):

$$q_a = \rho_r \cdot \frac{\Delta x}{2} \cdot \Delta y \cdot 1 \cdot c_{pr} \cdot \frac{T_{i,j,k+1}-T_{i,j,k}}{\Delta t}, \qquad (4)$$

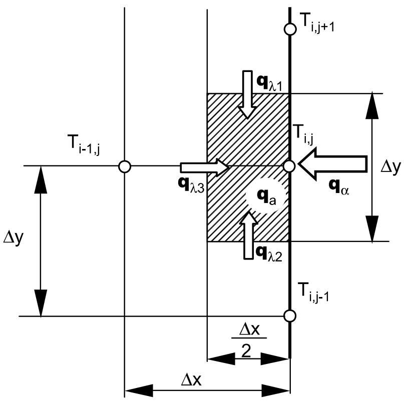


Fig. 2. Energy balance at the duct boundary in a selected node.

$$\left.\begin{aligned}
q_{\lambda 1} &= \lambda_r \cdot \frac{\Delta x}{2} \cdot 1 \cdot \frac{T_{i,j+1,k+1} - T_{i,j,k+1}}{\Delta y},\\
q_{\lambda 2} &= \lambda_r \cdot \frac{\Delta x}{2} \cdot 1 \cdot \frac{T_{i,j-1,k+1} - T_{i,j,k+1}}{\Delta y},\\
q_{\lambda 3} &= \lambda_r \cdot \Delta y \cdot 1 \cdot \frac{T_{i-1,j,k+1} - T_{i,j,k+1}}{\Delta x},
\end{aligned}\right\} \tag{5}$$

$$q_\alpha = \alpha \cdot \Delta y \cdot 1 \cdot (T_f - T_{i,j,k+1}). \tag{6}$$

Because of the application of a rectangular system of coordinates, nodes of the spatial net conformant with the condition of orthogonality of the heat stream density vector have been assigned to the nodes situated at the curved boundary of a duct.

Values of the heat transfer coefficients α at the surfaces of the rotating circular ducts, required for the calculations, were adopted based on the results of own research. The research was performed experimentally by means of electrolytic technique with the use of the analogy between mass and heat transfer. Detailed description of the research has been presented in [15,16]. The dimensionless equation obtained on the basis of experimental results has a form

$$Nu = 1.714 Re^{0.33} \quad \text{for } Ro = 0.1 \quad [16], \tag{7}$$

where the value of Rossby number results from the hydraulic characteristic of the real heat exchanger [8,9].

3.4. High-speed rotating heat exchanger as a distributed-parameter system

Processes of heat transfer occurring in exchangers are usually described by means of the following mathematical models [1]:

- *lumped-parameter model*, in which intensive parameters of the heat exchanging fluids and diaphragms are constant along the exchanger length;
- *lumped-parameter section model*, applying a partition of the exchanger into sections, in which assumptions conformant with the lumped-parameter model are being applied,
- *distributed-parameter model*, in which intensive parameters of the heat exchanging fluids and diaphragms are continuous functions of a spatial coordinate of the device in which the heat transfer processes occur.

The lumped-parameter models are described by means of ordinary differential equations with time derivatives, while the distributed-parameter models—by means of systems of partial differential equations with derivatives with respect to both time and spatial coordinates.

For the analyses conducted, a simplified version of the distributed-parameter model has been adopted for description of the heat transfer processes in a heat exchanger.

The intensive parameter of the temperature of the partition or of the rotor, which is the case in a high-speed rotating exchanger, was a function of the spatial coordinate. The temperature of the heat exchanging

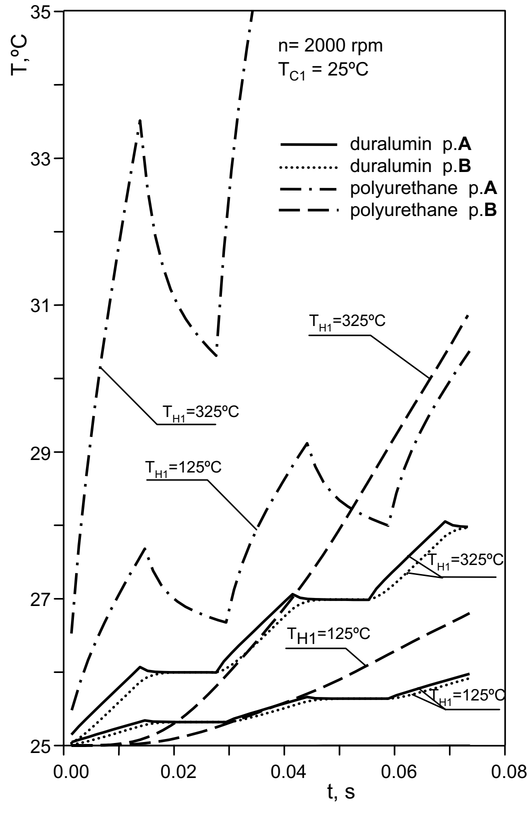


Fig. 3. The initial phase of thermal start-up.

fluid (air), calculated on the grounds of the energy balance equations in respective segments along the duct length [11,13], was an implicit function of a spatial coordinate, although it did not follow directly from the application of the partial differential equations for the description of heat transfer in the fluid.

4. Results of the calculations

The considerations presented in the previous sections allowed for a determination of temperature distribution patterns in the rotor material in the course of thermal start-up, i.e. when the transient processes are most inten-

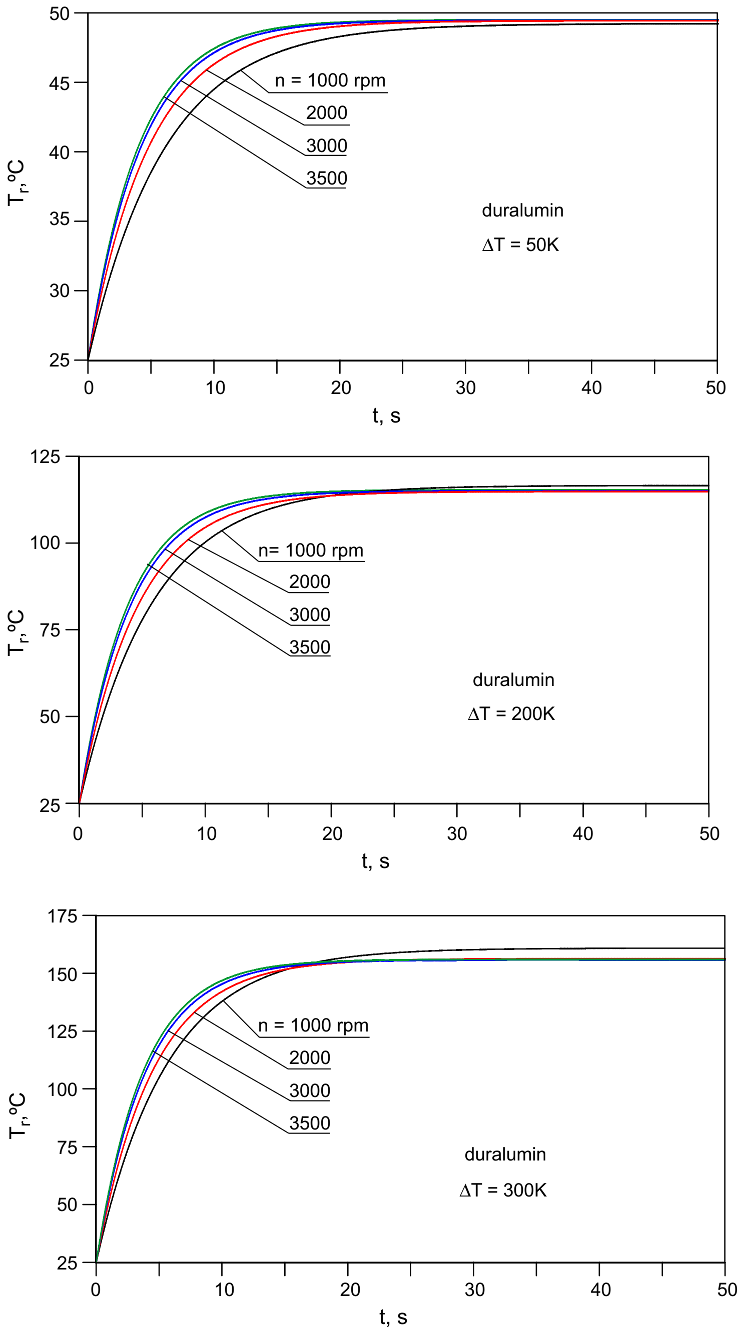


Fig. 4. The effect of the rotational speed of the rotor on rotor temperature during the period of thermal start-up.

sive. The adoption of various rotational speeds, gas temperatures at the inlet and materials made it possible to determine the effect of these parameters on the changes of the temperature as a function of time. With the use of these results, graphs were elaborated representing the dependence of the rotor's dimensionless temperature on the Fourier number, with the Biot number used as a parameter.

4.1. Initial phase of thermal start-up

The following procedure has been applied to obtain the changes of rotor temperature:

- the first initial condition $T_0 = T_{C1}$ has been valid,
- Eq. (1) has been applied in the area S (Fig. 1),
- boundary condition of the third type has been valid,
- constant temperature of the hot gas at inlet and constant rotational speed of the rotor has been assumed,
- Eq. (1) has been solved using the finite difference method (system of equations (2)); the temperature distribution in the element S after the first passing of the rotor through the region of the rotor heating has been obtained,
- the procedure has been repeated for the following sages of the rotor cooling and heating.

The calculations has been made until the obtained temperature distribution is repeated periodically in the heating and cooling regions.

Examples of plots of temperature variations at the duct surface (point A in Fig. 1) and within the material (point B) are presented in Fig. 3. The data are related to the initial phase of thermal start-up, at the rotor's speed equal to 2000 rpm. A rotor made of duralumin and of polyurethane has been considered. The temperatures of the hot gas at the inlet equal to 125 °C and 325 °C, respectively, have been adopted.

Examples of the dependence of the rotor temperature (mean from area S) on rotational speed are presented in Fig. 4. The ese are cases concerning the rotor made of duralumin and the temperature difference of the inlet gases $\Delta T = T_{H1} - T_{C1}$ equal to: 50, 200, 300 K (T_{C1} = 25 °C). Fig. 5 presents the dependence of the rotor temperature on the rotor material.

Similar patterns of the rotor material temperature variation in the period of time before reaching the quasi-stationary temperature for other cases presented in Section 2, served as a base for construction of the plots in dimensionless form.

4.2. Rotor heating during thermal start-up—a dimensionless form

The dimensionless temperature has been defined as

$$\Theta_r = \frac{T_r - T_0}{T_m - T_0}, \tag{8}$$

where T_0—rotor initial temperature (before the start of the heating process); $T_m = (T_{H1} + T_{C1})/2$; T_r—average temperature of rotor material.

The characteristic dimension L occurring in the expressions of the Biot number and the Fourier number has been considered as the ratio of the rotor material volume to the heat transfer surface. In the case of a rotor with radial ducts of a constant circular cross-section applied here, one obtains

$$L = \frac{V - V_p}{A_i} = \frac{(1-p)\cdot d}{4p}, \tag{9}$$

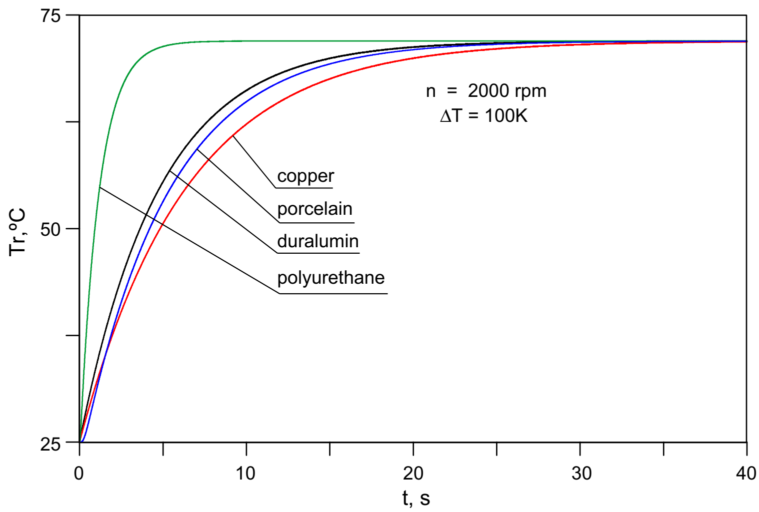


Fig. 5. The effect of rotor material on rotor temperature during the period of thermal start-up.

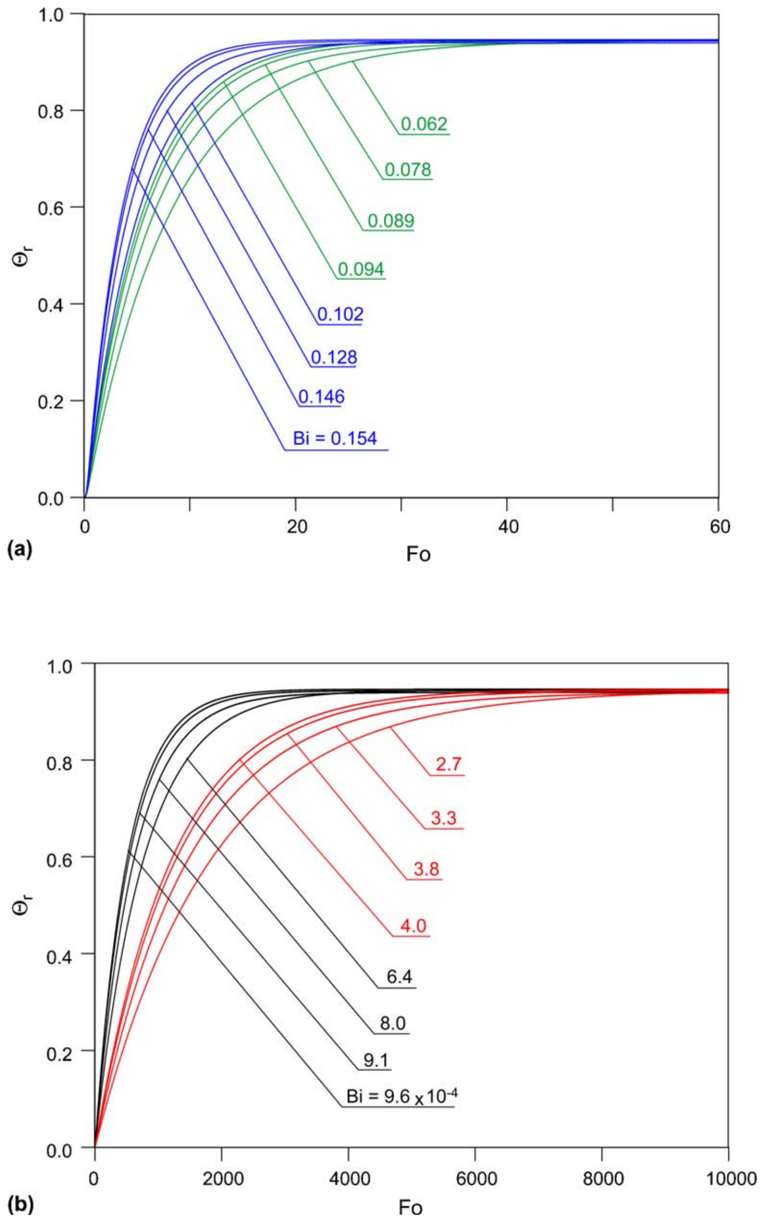


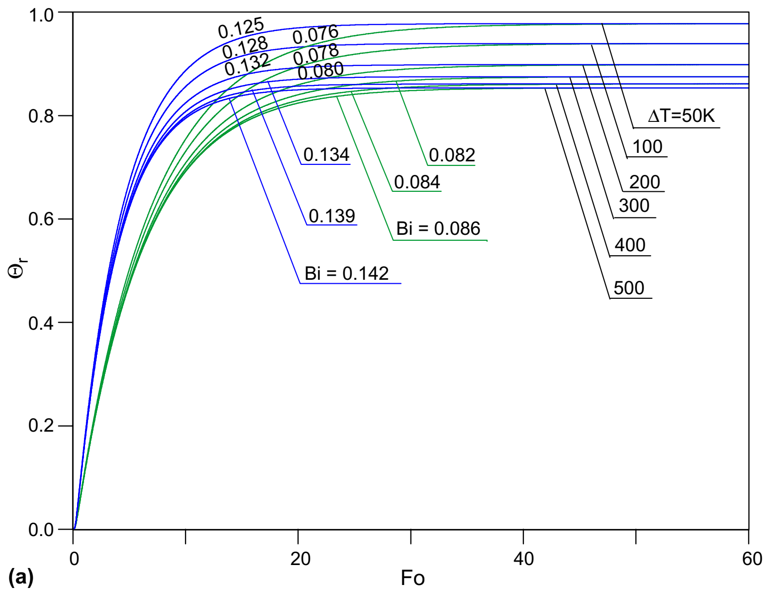
Fig. 6. Dependence of the mean dimensionless rotor temperature on Fourier number during the period of thermal start-up at: (a) $Bi \in \langle 0.062, 0.154 \rangle$, (b) $Bi \in \langle 2.7 \times 10^{-4}, 9.6 \times 10^{-4} \rangle$.

where V—total volume of rotor, V_p—total volume of pores (ducts), A_i—inner surface of ducts.

For the rotor under consideration, $L = 0.71 \times 10^{-3}$ m.

Graphs representing the dependence of Θ_r on Fo, presented in Fig. 6a, have been created with the use of the calculation of temperature distribution patterns for rotors made of: copper—dashed line, duralumin—solid line, and in Fig. 6b of: polyurethane—dashed line, and porcelain—solid line. The following temperatures of the gases at the inlet were applied: $T_{C1} = 25$ °C, $T_{H1} = 125$ °C, and the rotor's velocities of 1000, 2000, 3000, and 3500 rpm.

The Biot number values follow from the values λ of the assumed rotor material and the values α varying with a rotational speed. As for Fig. 7a and b, they have been



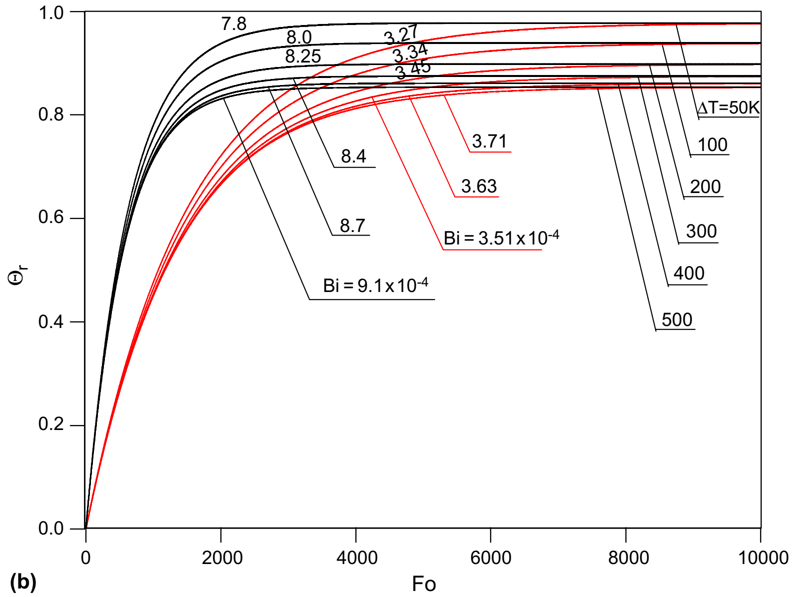


Fig. 7. Dependence of the mean dimensionless rotor temperature on Fo and ΔT during the period of thermal start-up at: (a) $Bi \in \langle 0.076, 0.142\rangle$, (b) $Bi \in \langle 3.27\times 10^{-4}, 9.1\times 10^{-4}\rangle$.

created with the following assumptions: rotor materials as above, rotational speed $n = 2000$ rpm, and the temperature difference of the inlet gases $\Delta T = T_{H1} - T_{C1}$ equal to 50, 100, 200, 300, 400, and 500 K ($T_{CI} = 25$ °C).

In this case, the Biot number values are dependent also on the variations of the rotor's material thermal conductivity λ_r and on the variations of α being a function of the Nu number, and as a consequence, on gas thermal conductivity λ_g depending on temperature.

In Fig. 8a, a summary of the results included in Fig. 6a and b and, in Fig. 8b, a summary of the results included in Fig. 7a and b respectively are presented in a double logarithmic system of coordinates. It follows from a comparison of Fig. 8a and b that the curves presented in Fig. 8b, created with assumptions different than those in Fig. 8a, constitute their averages, which may serve as a proof of correctness of the dimensionless results obtained.

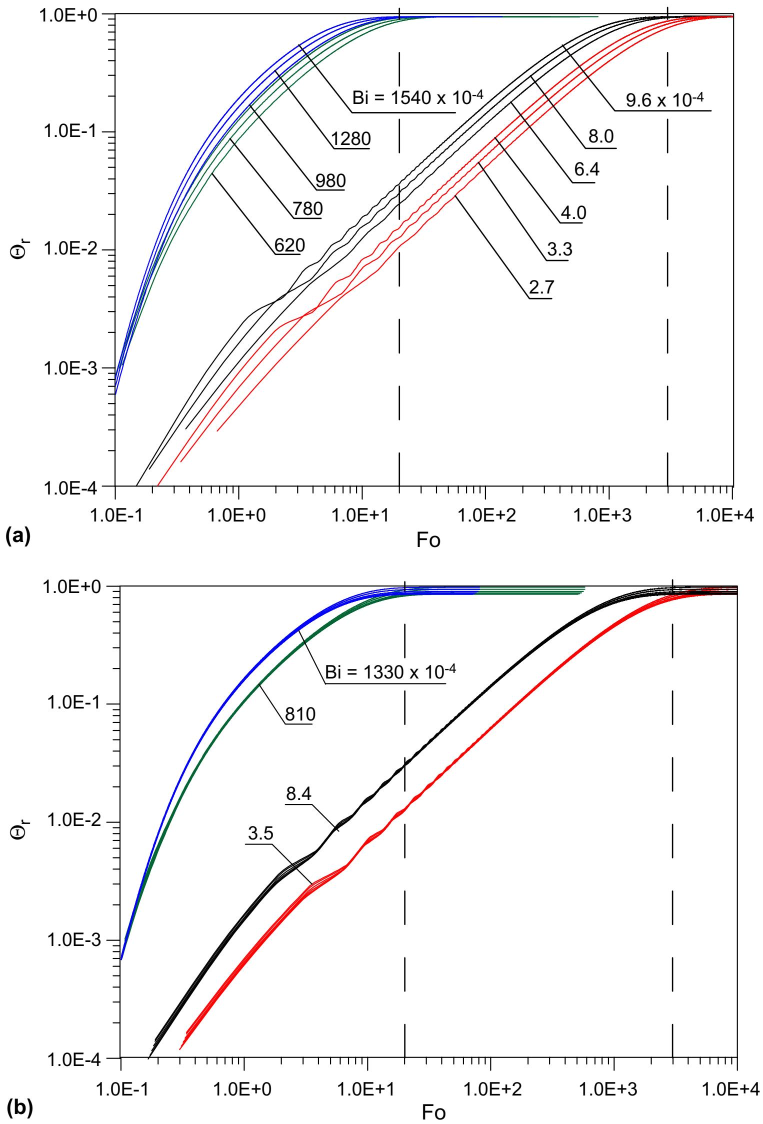


Fig. 8. Graphs for determining average temperature of the rotor of high-speed rotating heat exchanger during the period of thermal start-up (a) $Bi \in \langle 2.7\times 10^{-4}, 1540\times 10^{-4}\rangle$, (b) for averaging Bi number.

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